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# The ‘Indian rod trick’ via parametric excitation

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A seemingly paradoxical experiment is described whereby a length of wire is stabilised upside down by vertical periodic oscillation of its support. The quantitative details of the experiment reveal an upper and lower bound on the excitation frequency for stability. The results of recent theories are presented that explains the details of what is observed. It relies on a new phenomenon of so-called resonance tongue interaction. The result is verified via asymptotic calculation based on a one-dimensional rod model and numerical results on a spatially discretised system of links. This novel gravity defying effect has potential application to the stabilization of other spatially extended systems via parametric excitation.

**Keywords:** rod mechanics, experimental dynamics, parametric excitation

There has long been a fascination with phenomena that defy gravity, such as the legendary Indian rope trick and various more reputable scientific phenomena (Mahadevan et al. 1998, Dullin & Easton 1999, Thomsen & Tcherniak 2000). For example, it has been known since 1908 (Stephenson 1908) that rapid, small-amplitude, parametric (i.e. vertical) harmonic excitation can balance a single pendulum in an upside down position. Several authors, e.g. (Otterbein 1982), have wondered whether the Indian rope trick may be performed this way. Indeed it was demonstrated (Acheson & Mullin 1993) and proved mathematically (Acheson 1993) that a chain of  $N$  linked pendulums may be stabilised upside down by high enough frequencies. Unfortunately, in the limit of a rope, i.e.  $N \rightarrow \infty$ , the required frequency tends to infinity. It was reported (Acheson & Mullin 1998) that a rope with bending stiffness, namely a wire, can be stabilised at finite frequency, but no theory of why it works nor any experimental results have been published. This Letter presents quantitative details and the explanation of an experiment on an upright wire, just long enough to fall over under its own weight. It will show there are upper and lower bounds on the excitation frequency to perform this gravity defying ‘Indian wire trick’.

The experimental set up was as follows. A length of intrinsically straight domestic ‘curtain wire’, which consists of a tightly wound steel spring of approximately 3mm diameter clad with a 0.5mm plastic coating, was held upright by a clamp. This was driven vertically using a sliding crank device to supply approximately sinusoidal excitation. The amplitude of this oscillation was held fixed at  $\Delta = 2.2\text{cm}$

peak to peak. The length of  $\ell$  wire beyond the clamp was allowed to vary, as was the frequency of excitation  $\omega$  within a range 0–35Hz. It was found that the longest length of wire which was able to support its own weight was  $\ell = \ell_c = 55.3\text{cm}$ . Longer lengths resulted in catastrophic collapse to a buckled state. This bifurcation is sub-critical as the buckled states are of large amplitude to the extent that the tip of the wire is beneath the support as shown in Fig. 1(a) (cf. the experiments by Benjamin reported in (Iooss & Joseph 1990)).

Taking a fixed length  $\ell$  a little greater than  $\ell_c$ , initially held loosely upright, and switching on the parametric excitation it was found that the vertical position was made stable for a range  $\omega_1 < \omega < \omega_2$ . (see Fig. 1(b)). This stability region in the  $(\ell, \omega)$  plane is depicted in Fig. 2(a), which was obtained by taking a fixed  $\ell$  and making a slow sweep in frequency allowing time for transient effects to decay. The lower stability boundary  $\omega_1(\ell)$  is characterised by the rod falling over upon reduction of  $\omega$ , rather like the static buckle of the unforced problem. This falling is typically preceded, upon decreasing  $\omega$  by a definite lean (Fig. 1(c)). The upper instability at  $\omega_2(\ell)$  is more dramatic. As the frequency is increased, the rod suddenly becomes unstable to planar oscillations which are violent, uncontrollable and quite often cause permanent deformation to the helical coil. At the onset of the instability, the oscillations are at the same frequency as the vertical drive and are predominantly in the third vibration mode; Fig. 1(d). The quantitative results presented in Fig. 2(a) represent, for each  $\ell$ -value, the average over several runs, with care being taken to use new specimens of wire whenever there was deformation. Despite the care, evidence was found of additional resonance tongues within the stability region. That is, lateral oscillations were sometimes excited depending on transient effects. For the majority of the stability region though, the wire was found to be stable such that the tip could be given a slight push and the resulting lateral motion would decay. A repeatable exception to this was for  $\omega$  just above  $\omega_1$ , where a small amplitude circular motion of the tip was often excited. This motion is an order of magnitude slower than the vertical excitation and had a tendency for its frequency to decrease to zero as  $\omega$  was decreased to  $\omega_1$ .

The same phenomenon of stabilization beyond the critical length for buckling was also found to be possible using other materials such as niobium wire, but the curtain wire results were found to be the most robust. Let us now turn to a theoretical explanation of these results.

A simple theory (Acheson 1993) predicts the stabilization upside down of a system of  $N$  jointed pendulums without bending stiffness (verified experimentally in (Acheson & Mullin 1993)). These results do not directly apply here since taking  $N \rightarrow \infty$  results in a string with no bending stiffness and a prediction that the required frequency of excitation would be infinite. Instead, it is useful to consider a continuum model in which the wire is modelled as a linearly elastic rod with bending stiffness  $b$  (Champneys & Fraser 2000). In the absence of damping, and ignoring geometric nonlinearity, the lateral displacement  $u$  at arclength  $s$  along the rod ( $s = 0$  being the clamped end) is governed by the dimensionless equation

$$\eta \ddot{u} + (1 - \eta \varepsilon \cos t)[(1 - s)u']' + bu'''' = 0. \quad (0.1)$$

Here a prime represents differentiation with respect to the rod's arclength, a dot

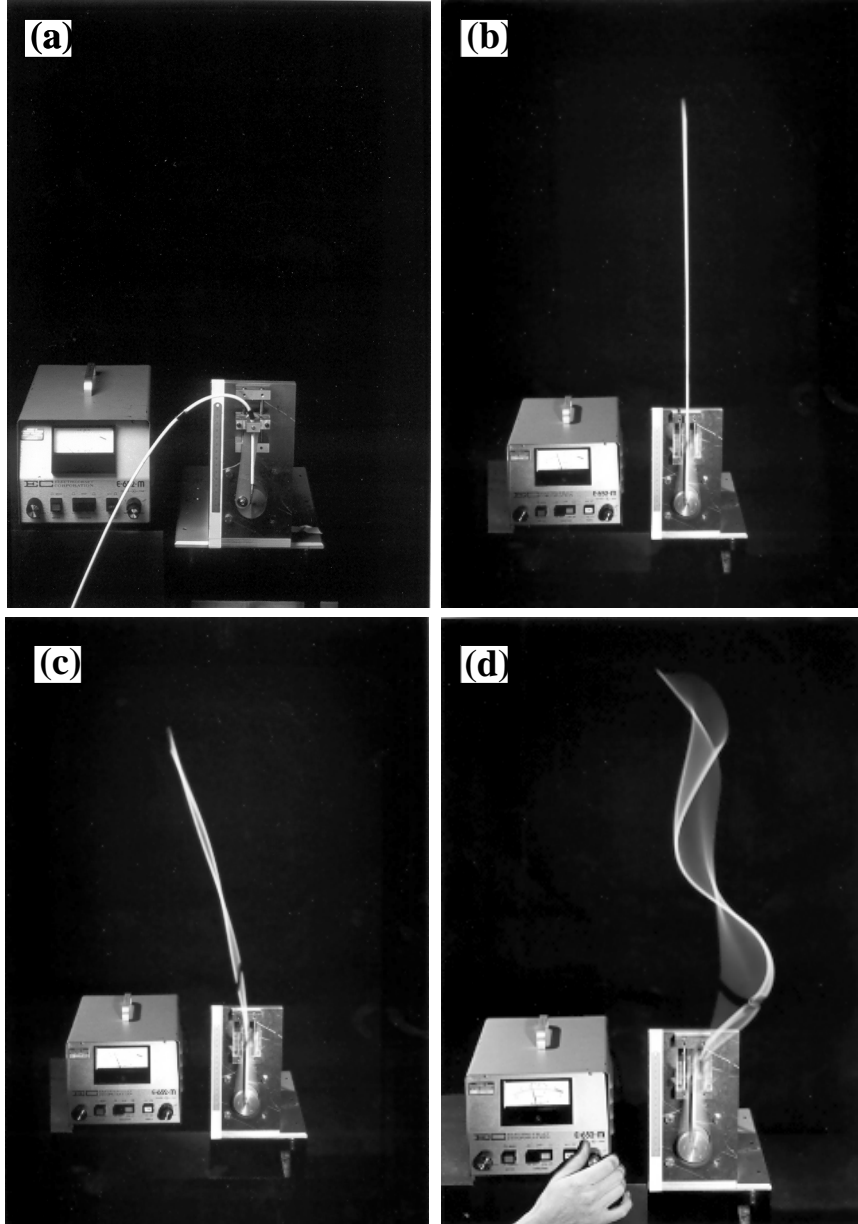


Figure 1. The experimental demonstration: (a) the sagging buckled wire, (b) the stabilized vertical wire, (c) leaning motion near the lower-frequency ‘falling over’ instability, and (d) motion near the higher-frequency dynamic instability.

differentiation with respect to time and the dimensionless parameters are

$$\eta = \frac{4\pi^2\omega^2\ell}{g}, \quad \varepsilon = \frac{\Delta}{\ell}, \quad B = \frac{b}{mg\ell^3};$$

$\eta$  and  $\varepsilon$  are the dimensionless frequency and amplitude of the excitation and  $B$

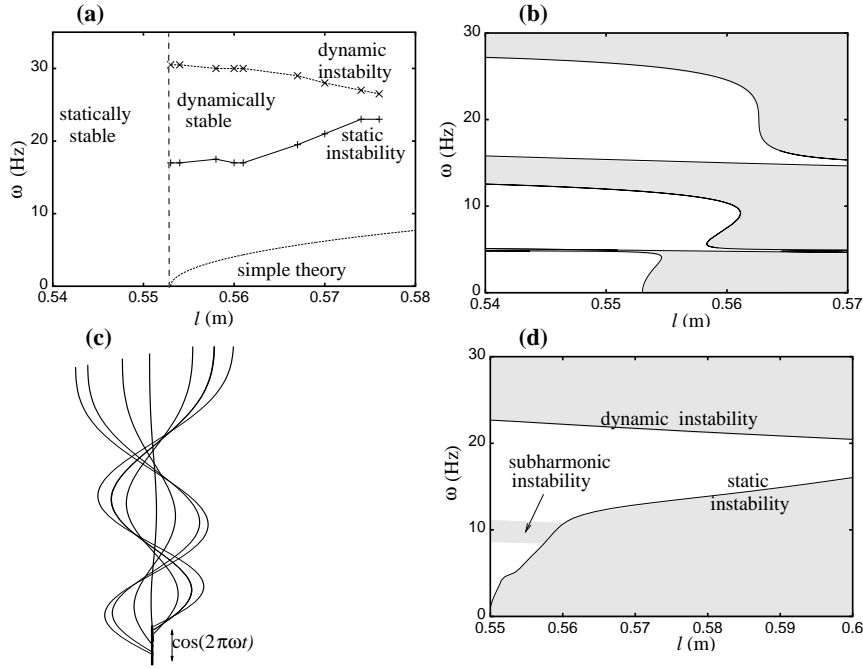


Figure 2. Quantitative details of the experimental results and various theories. See text for detailed explanation. (a) Experimental data and the ‘lower bound’ (2) for stability. (b) Resonance tongue interaction asymptotic results. (c) The shape of the harmonic resonance of the 3rd-spatial mode. (d) Numerical results on the stiff, damped linked pendulum model.

is the scaled bending stiffness. By analysing the fundamental instability in this model using asymptotic methods, a simple dimensionless formula was derived in (Champneys & Fraser 2000) for a lower bound on the excitation frequency for stability of the upright rod

$$\omega^2 > 1.27 \frac{\ell g}{4\pi^2 \Delta^2} \left( 1 - \frac{\ell_c^3}{\ell^3} \right). \quad (0.2)$$

This lower bound is plotted for the experimental values of  $\ell_c$  and  $\Delta$  as a dashed line in Fig. 2(a). Note that it is a gross underestimate! Also, it does not predict the upper stability boundary, which according to the theory could be caused by any one of an infinite possibility of dynamic resonances.

We shall present here the results of a new double-scale asymptotic analysis of Eq. (1) the details of which will appear elsewhere (Fraser & Champneys 2001). These results reveal a hidden subtlety in this problem caused by the interaction between certain harmonic resonances and the pure falling-over instability. This new phenomenon of *resonance tongue interaction* has striking consequences that explain the two forms of instability seen in the experiment and the qualitative shape of the stability region in the  $(\omega, \ell)$ -plane. The analysis is valid in the asymptotic limit  $\varepsilon \rightarrow 0$ . It shows how at certain special values  $\eta_c$  of the dimensionless frequency  $\eta$  the two instabilities occur at the same value of the third dimensionless parameter  $B$ . In the case that one of the instabilities is the excitation of any one of the vibration

modes of the rod at the frequency of the drive and the other is the pure falling over, then the coefficients of the quadratic terms in the asymptotic description of the instability boundaries become infinite. This in turn causes an increase in the parameter range leading to stability for  $\eta$ -values just less than the critical  $\eta_c$ , and the vanishing of the stability region for  $\eta > \eta_c$ . Just such a critical interaction is found to occur for the 3rd-spatial mode at  $\eta = 1813.5$ . This corresponds to  $\omega = 28.5\text{Hz}$  for the experiment on curtain wire. The harmonic response of the third spatial mode of the wire according to the theory is depicted in Fig. 2(c). Note its similarity with the experimentally observed dynamic instability (Fig. 1(d)).

The full results of the asymptotic analysis for the experimental values of  $\ell_c$  and  $\Delta$  are plotted in Fig. 2(b), the shaded parameter regions correspond to where this analysis predicts instability. Note the ‘dolphin’s nose’ shaped stability region which bears a strong resemblance to the experimental stability region in Fig. 2(a). Other zones of instability within this stability band are also predicted using similar asymptotic theory (results not depicted), and correspond to tongues of sub-harmonic resonance of the second and third vibration modes. It is worthwhile also to note that the asymptotic methods predict circular oscillations with frequency tending to zero as the falling over instability is approached, just as observed in the experiments.

Finally, in order to test the theoretical results numerically and also to include material damping which is evidently present in the experiment, we have derived a model consisting of  $N$  identical rigid pendulums coupled by stiff, damped linear springs. By correctly scaling the mass and length of each pendulum, the limit  $N \rightarrow \infty$  approaches that of the continuum model (0.1) with an additional damping term  $\Gamma \dot{u}''''$  on the left-hand side. Here  $\Gamma$  is a dimensionless parameter representing internal material damping. By using numerical bifurcation techniques (Doedel et al. 1997) we have been able to plot out boundaries of stability in parameter space for  $N$  up to 32. A striking convergence is found with increase of  $N$ . The results for  $N = 16$  and  $\Gamma = 0.004$  and other parameters as for the experiment are plotted in Fig. 2(d), with again shading representing instability. The same characteristic shape of the stability boundary is recovered, with a small region of sub-harmonic instability of the second vibration mode. Note the quantitative agreement with the experiment apart from a small down-shift in the frequencies, presumably due to an overly simplistic model of material damping.

In conclusion, this Letter has revealed for the first time the experimental details of a counter-intuitive mechanical phenomenon and provided a convincing theoretical explanation. The theory, which relies on detailed calculations presented elsewhere, shows how a novel interaction between two instability mechanisms leads surprisingly to stability. The theory, which is valid only for asymptotically small amplitude excitation, has been supported by numerical bifurcation analysis of a carefully discretised model which is valid for arbitrary excitation amplitudes. Note, however, that neither form of modelling takes into account the particular (nonlinear) material properties of the curtain wire used in the experiment. Hence the results should be relevant to a broad class of continuously flexible structures. Moreover, the new mechanism of creating stability via resonance tongue interaction has potential application across the physical sciences.

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